On the Possible Detection of Tsunamis by a Monitoring of the Ionosphere

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It has been suggested (Hines, 1972) that the atmospheric gravity waves that are generated by a tsunami may well produce an identifiable ionospheric signature that could be employed for purposes of tsunami warnings. The intrinsic difficulties to be faced by this suggestion derive from the geometrical spreading of the tsunami signature, from the possible occurrence of heights of null response, from the reflection and absorption of wave energy in the regions between the ocean surface and the ionospheric height of observation, from the time delay experienced by the tsunami signature in reaching that height, from a degradation of amplitude when the signature is detected via its effect on isopleths of electron concentration, and from the competition of 'noise' that would obscure the signature. These difficulties are assessed in the present paper and are found to be of only marginal consequence to the original suggestion, which is therefore reinforced as a proposal for an operational system.

INTRODUCTION

Tsunamis, the 'tidal waves' of popular literature, often cause considerable destruction when they impact on coastal regions and coastal communities. For this reason, the detection of their generation and propagation in time to give warnings is a matter of considerable concern. Attempts at such detection are hampered, however, by the relatively small amplitudes of vertical displacement they produce (at most a few tens of centimeters) in the surface of the open ocean, coupled with their relatively long horizontal wavelengths (hundreds of kilometers); the tsunami 'signal' at the ocean surface is simply lost in the 'noise' produced by the chop and swell of more random and more innocuous wave systems.

Most past efforts in the search for a solution to the problem of prediction have centered upon seismological measurements, on the grounds that most tsunamis are produced by earthquakes whose occurrence can be readily detected and located. These efforts have intrinsic limitations, however, if only because the relationship between earthquake occurrence and tsunami generation is by no means a simple one. It involves not only such obvious factors as magnitude and location, but also complexities (such as the type of fault movement) that can be inferred from the presently available seismic data only after extensive analysis. In consequence, the seismic approach seems to be limited at best to a prediction of the probability of tsunami occurrence on the occasion of any given earthquake. Not only is this limited capability inadequate to the need, but it carries with it the dangers of adverse response that follow upon the inevitable false alarms. There is a current trend therefore to develop techniques whereby a tsunami may be detected directly, after it has been generated but before its harmful effects are felt. One method, currently proceeding to preliminary test, would make use of pressure sensors deployed over the ocean floor and so would detect the tsunami by means of its effects well below the ocean surface. The present paper explores theoretically a second method, one that would go to the opposite extreme of detecting the tsunami's effects well above the oceanic surface, within the ionosphere at heights of 125 km or so.

The basis of the proposal, which incidentally suggested itself as a by-product of the great Alaskan earthquake of 1964, has been outlined already [*Hines*, 1972]. It combines three elementary facts: a tsunami must displace the atmosphere as it propagates across the open ocean, the atmosphere responds to this excitation by propagating gravity waves obliquely upward, and these waves grow nearly exponentially with height as they proceed into the rarefied regions of the upper atmosphere. An elementary evaluation of the growth effect leads to an amplification factor of about 10^4 by the time the 125-km level is reached. A vertical displacement of several centimeters at the ocean surface might then be expected to give rise to displacements of several hundred meters in the ionospheric E region. This might be degraded somewhat on conversion to ion contour displacement but should still be detectable with the aid of conventional (e.g., phase height) ionosonde equipment.

Three principal processes act to offset the intrinsic growth: geometric spreading of the atmospheric wave energy, dissipation of this energy within the atmosphere, and partial reflection of it by background wind and temperature structure beneath ionospheric levels. These processes, the complications imposed by conversion of the tsunami signature from the neutral gas to the ionization, and the possibility that the signal would be subject to too long a delay in reaching the ionosphere to be useful are examined in the following pages. We conclude in the end that a detectable signal is likely to be available, whereby an array of ionosondes, linked in real time to a central analysis station, could identify dangerous tsunamis before they do damage.

IDEALIZED ATMOSPHERIC RESPONSE TO AN IDEALIZED TSUNAMI

The possibility of severe geometrical spreading of the atmospheric wave energy launched by a tsunami initially appeared to us to be the most serious of the potential causes of signature degradation. We examine it first, by way of an idealized calculation that employs an idealized tsunami.

The geometrical spreading will, we believe, arise primarily in directions perpendicular to the leading edge of the tsunami. This belief is based upon the high coherency that may be expected to be obtained along the leading edge for distances that greatly exceed typical horizontal wavelengths in the direction of propagation, a coherency whose existence appears to be confirmed by recent calculations [*Hwang et al.*, 1972] and whose effect would be to suppress any divergence of atmo-

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spheric wave energy sideward. We are then led to adopt a two-dimensional approach to the problem, one in which the tsunami is represented by a vertical z displacement of the lower boundary of the atmosphere, and this displacement varies with and propagates in a single horizontal x direction only.

The temporal variation of a representative tsunami is shown in Figure 1 from the work of *Van Dorn* [1969], who obtained the record from one of a series of tide gages off Wake Island following the great Alaskan earthquake of 1964. The initial oscillations constitute the nondispersive 'Jeffrey's phase,' which is followed in due course by a dispersive train of waves, it being only at the head of the tsunami that the phase speed of the wave is given by the (nondispersive) shallow water speed. Because of contamination of the wavelet after the first few cycles, only the initial part of the wave form will be sufficiently coherent for great distances along the wave front to act as an efficient source of atmospheric waves for our purposes. For present modeling, then, it is sufficient to represent the tsunami by a short nondispersive pulse comprising at most two or three strong surface undulations.

We have in fact adopted the idealized wave form depicted in Figure 2, which was constructed from the first few cycles of the Airy function [*Abramowitz and Stegun,* 1965] multiplied by a Poisson distribution. The scale of the wave form was taken in our subsequent computations to be such that one unit of x in Figure 2 represents 100 km, which implies dominant horizontal wavelengths of the order of 400 km. This value, when combined with a shallow water speed of 200 m/s, which is representative of the Pacific Ocean and which we assumed in the computations, implies dominant wave periods of the order of 33 min. That a wave form such as this should simulate the open ocean wave form of a tsunami follows from the theoretical discussion of *Stonely* [1963]; that it succeeded, at least crudely, can be seen from a comparison of Figures 1 and 2.

This idealized model tsunami was taken to underlie an idealized nondissipative model atmosphere, one that is stationary and isothermal except for the perturbations that are induced by the passage of the tsunami itself, which occurs at speed v in the $-x$ direction. With these idealizations the problem was reduced to that of wave generation in an idealized atmosphere blowing uniformly over a (two-dimensional) mountain range at speed v in the $+x$ direction, except for the obvious transformation of horizontal coordinates. The steady state solution to this problem can be obtained by application of the work of *Lyra* [1943] and/or *Queney* [1947, 1948] in the approximation that the Coriolis force is neglected (since the relevant wave periods here are small in comparison with half a day). Briefly, the unperturbed ocean-atmosphere interface $z = 0$ is perturbed by a deformation Z that is expressed in terms of its Fourier transform $\mathfrak{z}$:

$$Z(x + vt, 0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dk_x \cdot \exp\,[ik_x(x + vt)]\mathfrak{Z}(k_x, 0) \qquad (1)$$

where

$$\mathfrak{Z}(k_x, 0) = \int_{-\infty}^{\infty} dx \exp\,(-ik_x x)Z(x, 0) \qquad (2)$$

The steady state vertical displacement at height z above the unperturbed ocean-atmosphere interface is then found to be

$$Z(x + vt, z) = \frac{\exp\,(z/2H)}{2\pi} \int_{-\infty}^{\infty} dk_x \cdot \exp\,[ik_x(x + vt)]\mathfrak{Z}(k_x, z) \qquad (3)$$

where

$$\mathfrak{Z}(k_x, z) = (k_x, 0) \exp\,[i(-T_v)^{1/2}z/c] \qquad (4)$$

$$0 < k_x v < \omega_1 \qquad k_x v < -\omega_2$$

$$\mathfrak{Z}(k_x, z) = (k_x, 0) \exp\,(-T_v^{1/2}z/c) \qquad (5)$$

$$\omega_1 < |k_x v| < \omega_2$$

$$\mathfrak{Z}(k_x, z) = (k_x, 0) \exp\,[-i(-T_v)^{1/2}z/c] \qquad (6)$$

$$-\omega_1 < k_x v < 0 \qquad \omega_2 < k_x v$$

Here

$$T_v = k_x^2(c^2 - v^2) - n_g^2 c^2/v^2 - n_a^2$$

$$n_g = (\gamma - 1)^{1/2} g/c = 1.88 \times 10^{-2}\ \mathrm{s}^{-1}$$

$$n_a = \gamma g/2c = 2.08 \times 10^{-2}\ \mathrm{s}^{-1}$$

and c is the speed of sound equal to 330 m/s, γ is the ratio of specific heats equal to 1.4, g is gravitational acceleration equal to 9.8 m/s², and ω_1 and ω_2 are the two positive roots ω of

$$2\omega^2 = n_a^2 + k_x^2c^2 \pm [(n_a^2 + k_x^2c^2)^2 - 4k_x^2c^2n_g^2]^{1/2} \qquad (7)$$

such that $\omega_2 > \omega_1$. The form (5) corresponds to evanescent acoustic gravity waves, and the sign of the exponent is chosen to make these waves attenuate as height increases. The forms (4) and (6) correspond to internal acoustic waves ($k_x^2v^2 > \omega_2^2$) and internal gravity waves ($k_x^2v^2 < \omega_1^2$), and the sign of the

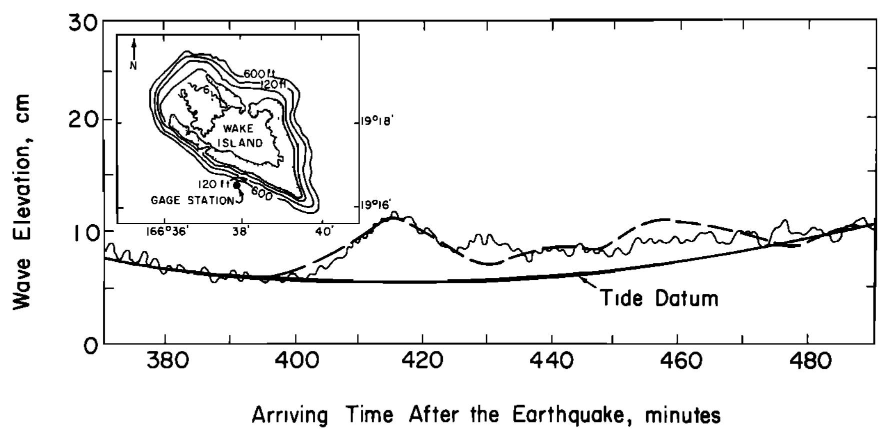

Fig. 1. The 1964 Alaskan tsunami observed at Wake Island by *Van Dorn* [1970] and compared with the prediction of the wave history by *Hwang et al.* [1972].

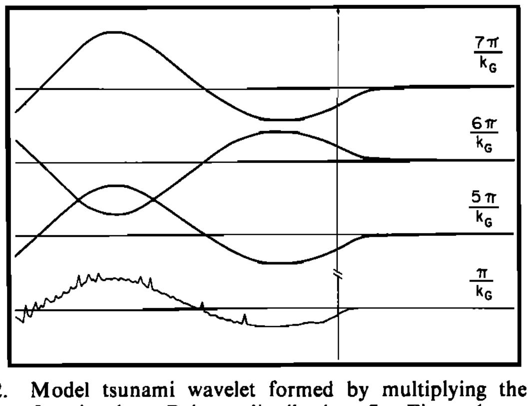

Fig. 2. Model tsunami wavelet formed by multiplying the Airy function by a Poisson distribution. See Figure 1.

exponent is chosen to make these waves radiate energy from the ocean-atmosphere interface upward. These choices are of course compatible with those derivable from an application of a causality criterion to the related transient problem [*Eliassen and Palm*, 1954; *Pierce*, 1965, Appendix B].

With $n_a > n_g$, as is the case, it may be confirmed that the condition $v^2 > n_g^2c^2/n_a^2$ would imply $k_x^2v^2 > \omega_2^2$, the condition $v^2 > c^2$ would imply $k_x^2v^2 > \omega_1^2$, and the condition $n_g^2c^2/n_a^2 < v^2 < c^2$ would imply $\omega_1^2 < k_x^2v^2 < \omega_2^2$. In the present application, only the first of these conditions is relevant. It requires the form (5) for all $k_x^2 < k_G^2$, the form (4) for all positive $k_x < k_G$, and the form (6) for all negative $k_x > -k_G$, where

$$k_G^2 \equiv (n_g^2c^2/v^2 - n_a^2)/(c^2 - v^2) \qquad (8)$$

and $k_G > 0$; $k_G = 8.75 \times 10^{-5}$ m^{-1} and $\lambda_G \equiv 2\pi/k_G = 72$ km in our model.

Our model tsunami, its dominant wavelengths being substantially in excess of 72 km, provides a $\mathfrak{z}(k_x)$ that is relatively small for $k_x^2 > k_G^2$. The contribution of the corresponding portions of the spectrum to the ionospheric signature is rendered even less important by the exponential attenuation implicit in (5). For all practical purposes, then, we need employ only (4) and (6), and we may limit the integration in (3) to the range $-k_G < k_x < k_G$:

$$Z(x + vt, z) = \frac{\exp(x/2H)}{2\pi} \int_{-kG}^{kG} dk_x \cdot \exp[ik_x(x + vt)]\mathfrak{z}(k_x, 0) \exp[\pm i(k_G^2 - k_x^2)^{1/2}z'] \qquad (9)$$

where $z' = z(1 - v^2/c^2)^{1/2}$ and the positive or negative option is to be taken as positive over the range $0 < k_x \leq k_G$ and as negative over the range $-k_G \leq k_x < 0$.

Our model tsunami leads to a $\mathfrak{z}(k_x)$ that tends to be relatively small when $|k_x| \simeq k_G$ and tends to become relatively large only for $|k_x| \ll k_G$. One might then attempt an approximation to (9) in which the second exponential is replaced by

$$\exp \pm(ik_Gz) = \cos(k_Gz) \pm i \sin(k_Gz) \qquad (10)$$

For heights $z' = z_G'$ such that $\sin(k_Gz_G') = 0$ and $\cos(k_Gz_G') = \pm 1$, the integration in (9) may be performed immediately:

$$Z(x + vt, z_G') = \exp(z_G/2H) \cos(k_Gz_G')Z(x + vt, 0) \qquad (11)$$

This result implies that the displacement caused by a tsunami, as measured at the succession of heights denoted by z_G', is simply mapped upward from the ocean surface, increasing in amplitude with height and being reversed alternately in sign; it implies no geometrical spreading of the tsunami signature whatever when that signature is measured at the special heights $z' = z_G'$.

This result is apparently well known to those who deal with the theory of mountain lee waves. It is illustrated for the particular case of a Gaussian bell-shaped mountain, for example, by *Queney* [1948] in his Figure 2, where the results of calculations that apply between the $z = z_G$ levels are also depicted. At the intermediate levels $z' \neq z_G'$, no simple evaluation analogous to (11) is available because of the reversal of sign on the right of (10), but Queney's illustrated computation indicates that the wave form changes smoothly from that appropriate to $\cos(k_Gz_G') = +1$ to that appropriate to $\cos(k_Gz_G') = -1$. There is some degradation of maximum signal amplitude at levels that lie between successive $z' = z_G'$ levels, with an accompanying horizontal dispersal of the signature, but the degradation is by a factor of no greater than 2 or 3 in his illustrations.

The approximation contained in (11), and the conclusion of small horizontal dispersal of tsunami signature to which it gives rise, cannot continue to be applicable indefinitely as height is increased. Since we require the information on geometrical spreading at great heights in the present problem, some further analysis appears to be appropriate.

A partial test for the validity of (11) at the heights $z' = z_G'$ might be made by requiring that the first-order phase correction in (9), namely, $k_x^2z'/2k_G$, should be small for the dominant k_x values at the $z' = z_G'$ levels:

$$z_G' \ll 2k_G/k_x^2 = \lambda_x^2/\pi\lambda_G$$

where k_x and λ_x are dominant values. In present circumstances, with dominant $\lambda_x \simeq 450$ km and $\lambda_G = 72$ km, this criterion would imply that the tsunami signature should suffer little horizontal dispersal at the $z' = z_G'$ levels that are well below an elevation of 900 km. The lower ionospheric levels where detection is anticipated, at elevations of 100–300 km, would appear to meet this condition, if only just.

In order that we might be certain of the point beyond a doubt the complete integral in (9) was evaluated numerically for the tsunami wave form depicted in Figure 2. The result of the evaluation is depicted in Figure 3 for the levels $z_G' = (1, 5, 6, 7)\pi/k_G$ or, for our model, at true heights $z = 45$, 226, 271, and 316 km. The somewhat erratic form of the lowest curve is a result of the computational technique and may be ignored. The anticipated reversal of sign between successive $z' = z_G'$ levels, the wave form being otherwise essentially intact, is worthy of note, however. Even more important for our purposes, the amplitude of the integral is found to remain undiminished at the upper levels (and, indeed, even increases slightly in the cycles shown); evidently, it is not attenuated by any effects of horizontal spreading. This establishes that the full exponential growth of wave amplitude with height is retained in the tsunami signature, at least at the $z' = z_G'$ levels, at ionospheric heights, despite our initial concern in the matter.

SUBSIDIARY CONSIDERATIONS

Since geometric spreading of the atmospheric wave energy in the direction of tsunami propagation does not result in a severe diminution of the tsunami signature at ionospheric *E* region heights, what we had anticipated as the most serious threat to the elementary exponential growth of the tsunami signature is seen to be no threat at all. We turn, consequently, to other processes that might interfere with the production of a useful signature. There are four such processes that appear to warrant consideration in this preliminary assessment: atmospheric wave dissipation, atmospheric wave reflection, delay of the onset time of the ionospheric signal, and conversion of the

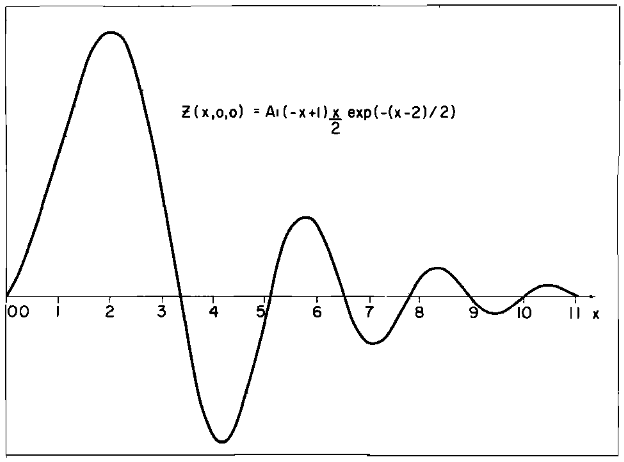

Fig. 3. Sample profiles of $Z(x')$ as produced by the translation at constant speed of the wavelet shown in Figure 2. See text for discussion of geometric details.

neutral gas signature into a detectable signature in the distribution of ionization.

A simple estimate of the attenuation introduced by dissipative processes can be made with the aid of equation (19) of *Pitteway and Hines* [1963]. Under the approximations appropriate to that equation and under the further valid approximation that their $\theta \gg 1$, it is found that a kinematic viscosity η alters the vertical wave number k_z from some nonviscous real value k_{z0} $(=(k_G^2 - k_x^2)^{1/2}(1 - v^2/c^2)^{1/2})$ to the complex viscous value $k_{z0} + \delta k_z$, where

$$\delta k_z \cong i\eta(\gamma - 1)^2 g^4 k_x^4/2k_{z0}c^4 \qquad (12)$$

which reduces to

$$\delta k_z \cong i\eta n_g^3/2k_x v^4 \qquad (13)$$

in present circumstances (i.e., when $k_{z0} \cong n_g/v = n_g k_x/\omega$). The isothermal n_g and its nonisothermal equivalent n_B vary with height in the real atmosphere, but both remain in the vicinity of 2.0×10^{-2} s^{-1} up to the ionospheric E region (with the exception of n_B in the troposphere; see below). With such values and other values previously cited,

$$\delta k_z \simeq 2i\eta \times 10^{-10} \qquad (14)$$

where δk_z is to be measured in units per meter and η in units per square meter per second. The attenuation to be expected at some height z_0 as a consequence of viscous dissipation below that level is then given approximately by the factor

$$\exp i \int_0^{z_0} \delta k_z \, dz \simeq \exp \left[-2 \cdot 10^{-10} \int_0^{z_0} \eta \, dz \right] = D \qquad (15)$$

The profile of kinematic viscosity below the 100-km level is not well established because of unknown and variable turbulence, but η is certainly no greater than 10^3 m^2/s over most if not all of that range. Above the 110-km level the profile is dominated by molecular kinematic viscosity, which increases to about 2×10^3 m^2/s at the 125-km level. The factor D for the 124-km level then amounts to something approximating exp (-0.03), which is to say that the corresponding reduction of wave amplitude at that level is about 3%. A similar reduction is to be expected as a consequence of thermal conduction [e.g., *Pitteway and Hines*, 1963], but clearly, neither reduction is serious for our purposes. For the tsunami parameters chosen here, in fact, the wave amplitude continues to increase with height almost up to the 200-km level, and even at the 250-km level it remains as large as it is at the 125-km level.

The foregoing estimates and their origin in the transition from (12) to (13) presuppose that the approximation $k_z \approx n_g/v$ applies throughout the height range of any significant dissipation. This would not be the case, even for our present model of a tsunami, if background winds beneath the ionosphere come close to matching the horizontal speed of tsunami propagation. Indeed, in the particular case of matched speeds the atmospheric wave energy would be almost completely quenched by processes associated with 'critical layers' [*Booker and Bretherton*, 1967; *Eliassen and Palm*, 1961; *Hines and Reddy*, 1967]. In fact, however, the background wind speed of the atmosphere is characteristically much less than typical tsunami speeds, except perhaps for rare occasions near the 105-km level. In normal circumstances the disparity is sufficiently great that the approximation is perfectly adequate for the purposes to which it has been put.

Reflection of atmospheric wave energy beneath the anticipated height of detection constitutes our secondary subsidiary consideration. This reflection is brought about in part as a result of height variations in the background winds just discussed and in part as a result of height variations of temperature. Some estimate of the effects of such reflection may be gained from the computations of *Hines and Reddy* [1967], in some cases as corrected by *Vincent* [1969]. These computations yield various transmission coefficients for various wave parameters for realistic profiles of wind and temperature both through the middle atmosphere and through the lower ionosphere. It is a rough but valid generalization to say that for horizontal trace speeds and wave periods such as those that characterize tsunamis, energy transmission coefficients ranged upward from 0.5 to 1. These values correspond to reductions of the transmitted wave amplitude by 30% or less. Though a 30% reduction would certainly be significant, it would not be sufficient to invalidate our general conclusions as to the detectability of the tsunami signature in the ionosphere. Further calculations, designed specifically to deal with the tsunami problem, might well be in order. If carried out, they should be extended to include propagation through the troposphere as well, since substantial reflection might be anticipated there. This reflection, if it occurs, would result from a lowering of n_B as a consequence of the thermal lapse rate of the troposphere. Over land, n_B is reduced to values typically of the order of 8–10 $\times 10^{-3}$ s^{-1}, which exceed the ω of tsunamis only by a factor of 2 or 3; over the ocean the reduction may not be as severe, and the reflection may not be of importance.

Whatever may be the detailed outcome of extended computations on the reflectivity of the middle atmospheric regions, one significant point may be made even at this stage: the reflected atmospheric wave energy is not absorbed but instead is reflected once again at the ocean surface. Ultimately, all that is launched out of the troposphere will escape into the upper atmosphere and so will contribute to the signal there. (It is clear that there will be no mode conversion into Lamb waves at the air-sea interface, since the horizontal trace speed of the component waves in the tsunami spectrum is invariant in the reflection process, and for all components of the spectrum this trace speed is equal to v rather than c, as would be required for Lamb waves.) The reflection process may tend to produce somewhat more spreading of the tsunami signature than our earlier computations suggest and may lower the peak amplitudes in the process, but it will not remove the energy from the signature as a whole. More than this we cannot say without a much fuller analysis than that undertaken here. Such an analysis has been initiated.

The third subsidiary consideration concerns the onset time of the tsunami signature in the ionosphere. This time will be determined in large part by the upward propagation of energy in the gravity wave portion of the spectrum ($\omega < n_g$). This propagation may be expected to occur with a speed given by the vertical component of packet velocity for freely propagating waves, which can be cast into the form $\omega k_z[\omega^2/c^2 - n_g^2 k_x^2/\omega^2]^{-1}$ quite generally in the absence of dissipation. In present circumstances, once again (i.e., $k_z \cong n_g/v = n_g k_x/\omega$ and $\omega^2/c^2 \ll n_g^2 k_x^2/\omega^2$ for the most important portion of the spectral content) this form reduces to $v^2 k_x/n_g$. With the insertion of numerical values cited previously the vertical speed of energy propagation is now found to be about 30 m/s. At this speed, energy would require a little over 1 hour to reach the 125-km level. After a time of that order it is to be expected that the steady state signature would rapidly take form as the observable deformation of the ionosphere at that height. Since we may be concerned with tsunami propagation times of the order of 4–15 hours, a delay of an hour or so in the onset of the signature at ionospheric levels would not seem to invalidate the ionospheric technique for the purpose of a warning system.

The fourth and final subsidiary consideration that we are prepared to touch on in this study is the conversion of the tsunami signature in the neutral gas to a signature in the distribution of ionization. Many factors affect this conversion, as will be apparent from a reading of papers such as *Hooke*'s [1968, 1969]. We shall deal only with the simplest of these, while acknowledging that a more thorough study would be wanted if the potential of an operational tsunami detection system were to be evaluated adequately.

The contemplated detection system might well employ for its warning signature the vertical motion of isopleths of electron concentration. A major, and in some circumstances dominant, contributor to this vertical motion is provided by the vertical motion enforced on the positive ions by the neutral gas, the electrons responding in whatever way they must to maintain charge neutrality. This vertical motion is given by [e.g., *MacLeod*, 1966]

$$V_z = (1 + r^2)^{-1}[r^2 u_z + r u_\eta \cos I + u_\xi \cos I \sin I + u_z \sin^2 I] \qquad (16)$$

where r is the ratio of collision frequency to gyrofrequency for ions, I is the magnetic dip angle (positive in the northern hemisphere), the ξ axis is directed southward (magnetically), the η axis eastward, and the z axis upward. Evaluation of the ratio V_z/u_z clearly depends on altitude via r, on azimuth of wave propagation via the disparate roles played by u_ξ and u_η, and on latitude via I. It depends also on the ratio $(u_\xi^2 + u_\eta^2)^{1/2}/u_z$, although for the waves of present interest ($\lambda_x \simeq 450$ km, $\lambda_z \simeq 90$ km) this ratio can be fixed at about 5.

In the E region, r decreases from about 4 near the 100-km level to about 1 near the 140-km level. Low in the E region, then, the first term or the first two terms in brackets in (16) will tend to dominate, and they provide contributions to V_z approximately equal to u_z in magnitude. High in the E region and in the F region the second or third term will tend to dominate and thereby provide contributions to V_z that are a few times u_z. The contributing terms can offset one another in certain circumstances and so can lead to a V_z that is in fact smaller than u_z, but these circumstances could not apply over the whole of the tsunami wave front; there could be isolated paths of advance of that front where the ionization signature of the tsunami would be relatively weak, but on other paths the ionization signature would be about as strong as if the ionization simply moved vertically with the neutral gas.

The further complications dealt with by *Hooke* [1968, 1969] which incorporate effects of electron convergence, electron production, and electron loss contribute further terms to the vertical motion of isopleths of electron concentration. These contributions are either no greater than the contribution of V_z, in which case the estimate of vertical motion given above remains valid (except for possible cancellations) or else they are greater, and the estimate must be increased. As we shall now discuss, the estimate given above is already adequate for our purposes in most circumstances. The only serious exception arises for the F region near the magnetic equator, where $\sin I \simeq 0$, when V_z is found from (16) to be much less than u_z. Figure 7 of *Hooke* [1968], which is drawn for wave parameters much the same as ours, reveals in this case that the effects of horizontal ion divergence are sufficient to make up for the loss of vertical ion motions in these circumstances, for they produce a vertical motion of isopleths of electron concentration that is just about equal to the vertical motion of the neutral gas.

FURTHER DISCUSSION AND CONCLUSIONS

At the ocean surface a potentially dangerous tsunami may have a vertical amplitude of oscillation of only a few centimeters. Allowing for the possible degradation of the ionospheric signature as a consequence both of wave reflection in the middle atmosphere and of conversion to a signature in the isopleths of electron concentration, we might take the effective vertical amplitude of oscillation to be only ±1 cm. This amplitude is subject to the full exponential growth expected of a gravity wave as it propagates from sea level to $z' = z_G'$ levels in the ionosphere, however, as we have now established, and that growth is given by the square root of the ratio of the atmospheric density at ground level to that at the ionospheric heights of interest. For the 125-km level the growth factor is almost exactly 10^4, and for the 250-km level it is almost exactly 10^5. We have found the dissipative attenuation to be negligibly small in the first case and to reduce the amplitude in the second case by a factor of about 10. Both in the E layer and in the F layer, then, we anticipate vertical movements of the isopleths with amplitudes of the order of ±100 m. Such amplitudes are comparable to the wavelengths of the radio waves that would be employed to monitor the isopleths and so are readily detectable in a phase height system of observation [e.g., *Vincent*, 1972]. Some degradation of amplitude may occur if the height of observation is nearly midway between neighboring $z' = z_G'$ levels, but by an amount that appears unlikely to invalidate this conclusion. The monitoring could in any event be conducted at two or more levels within the ionosphere, separated appropriately in height to ensure that at least one of them was located favorably for detection.

The wanted tsunami signal must of course be detected above the noise of other fluctuations. One possible source of such noise might be thought to be the random assortment of ocean waves that obscures the tsunami so effectively at sea level. If the random assortment of atmospheric waves to which it gives rise were to increase in amplitude exponentially with height as the tsunami signature does, it would obscure that signature just as effectively within the ionosphere. This does not happen, however, because the random assortment of ocean waves is characterized by periods that are well below the Brunt-Vaisala period of the atmosphere ($2\pi/n_g$) and by speeds that are less than c, a combination that gives rise only to evanescent atmospheric waves: (5) applies to the whole of the assortment and reveals a strong attenuation rather than growth of amplitude with height. The suggestion that the general spectrum of ocean waves might generate atmospheric waves that reach the ionosphere in strength [*Daniels*, 1952] has in fact already been dismissed on these grounds by *Eckart* [1953].

There remain for consideration the fluctuations that are imposed by other atmospheric waves, those that are generated by some meteorological or auroral process. Their potential role can be assessed only empirically, and this is best done at the sites where actual detection is contemplated. It is well known, however, that there are often fluctuations in the phase height of isopleths of electron concentration that exceed the 100-m amplitude we have estimated here for the tsunami signature [e.g., *Vincent*, 1972], and consequently, we must anticipate adverse conditions for signal detection much of the time.

The problem of signal detection is, however, greatly alleviated by the fact that potential tsunami sources are routinely identified by a presently existing seismic warning system. Given the origin of time and the location of a suspicious earthquake, the onset time of the ionospheric signature at a given location is predictable (although further calculation may be required), and its shape might be predictable as well. This sort of information is crucial to the detection of signals that are buried in noise in other systems of signal identification, and there is no reason to doubt that it could play a valuable role in the system contemplated here.

The possibilities are further improved by the fact that the system could employ an array of detectors, possibly mounted on weather ships or on a scattering of islands, feeding signals to a central analysis station following the identification of a potentially tsunamogenic earthquake. Correlative analysis of the various inputs, based once again on the known time and location of the earthquake in question, should go far to eliminate spurious signatures and to reveal clearly true tsunamis when these are of potentially dangerous strength.

Though we have identified phase height systems of ionospheric monitoring in this context, it should be remarked that different radio systems tend to detect preferentially different portions of the wave spectrum that are known to exist in the ionosphere. Possibly some other method of detection would be preferable for the identification of tsunami signatures.

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